Very special relativity as particle in a gauge field and two-time physics

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Abstract

The action for a $(3+1)$-dimensional particle in very special relativity is studied. It is proved that massless particles only travel in effective $(2+1)$-dimensional space-time. It is remarkable that this action can be written as an action for a relativistic particle in a background gauge field and it is shown that this field causes the dimensional reduction. A new symmetry for this system is found. Furthermore, a general action with restored Lorentz symmetry is proposed for this system. It is shown that this new action contains very special relativity and two-time physics.

1 Introduction

Lorentz symmetry has been the most important symmetry of physics for more than a hundred years. However, is very interesting to study systems with Lorentz violation at high energy, in fact some results about these systems are

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very attractive. For example, Hořava gravity breaks locally this symmetry, but it is power counting renormalizable [1]. Additionally, in quantum field theory extensions of the standard model without Lorentz invariance have been proposed [2, 3], some of them improve the behavior of Feynman diagrams [4]. Now, if Minkowski geometry is changed to another generalized geometry, Lorentz symmetry is changed too. Finsler geometry was recently put forward to replace Minkowski geometry and it is a background for several systems that break Lorentz symmetry [5]. In addition, Cohen and Glaslow showed that using space-time translations plus a special Lorentz subgroup it is possible to get a simulacrum of special relativity [6], this new theory was named Very Special Relativity (VSR). In VSR many of the consequences of Poincaré group are unchanged, but it leaves invariant some constant vectors, the so called "spurion fields". Cohen and Glashow suggested that VSR might be important at Planck scale. It is worth to mention that Finsler geometry is a good framework for VSR [7] and a relation between this theory and non commutative spaces was found [8, 9]. Furthermore, a generalized VSR was presented, where the usual line element is changed to [10]

$$ds^2 = \left(-dX^\mu dX_\mu\right)^{\frac{1-b}{2}} \left(-\alpha_\mu dX^\mu\right)^b, \tag{1}$$

here the vector α_μ is constant. This line element also was found in [11]. Some works about Lorentz violation can be seen in [12, 13, 14, 15, 16, 44, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28] and works about VSR can be found in [29, 30, 31, 32, 33, 34, 35, 36].

In this work it will be shown that massless particles in VSR only travel in effective (2+1)-dimensional space-time. This is a remarkable result, in fact there are some approaches to quantum gravity with dynamical dimensional reduction at Planck scale [37, 38, 39]. Furthermore we will find that the action for a particle in VSR can be written as an action of relativistic particle in a background gauge field, and this field causes the dimensional reduction. The symmetries of this action will be studied. Now, in order to understand breaking symmetry we can take two approaches, the first one is take a known system invariant under a symmetry group and then add terms which are not invariant under that group. The second one consisting in take a system without invariance under a group transformation and then impose that invariance. The second path is interesting, for example the action of free complex scalar field is not invariant under local $U(1)$ group, but if we

impose this symmetry we get complex scalar field in electromagnetic theory. Then, with the aim to restore Lorentz symmetry in VSR, we will take the second approach. In this approach, we will get a generalized action, which contains the action of VSR and two-time physics. It is worth mentioning that two-time physics acts like a model that unifies the dynamic of different systems [40, 41, 42, 43]. In particular, this theory contains the relativistic free particle and different particles in a curved space-time [43]. Namely, at the beginning we had a system with breaking Lorentz symmetry, and when this symmetry is restored a unified model is obtained. Then we can conjecture that if at Planck scale the Lorentz symmetry is broken, as long as it is restored a unified model is obtained too. A work about two-time physics and breaking Lorentz symmetry can be seen in [44]. It is important to mention that recently optical metamaterials with two-time properties have been proposed [45].

This manuscript is organized in the following way: In section 2 it is shown that the action for a particle in VSR is equivalent to a relativistic particle in a background gauge field; in section 3 Lorentz symmetry is restored; in section 4 the two-time action is obtained and finally in section 5 our summary is given.

2 Very special relativity as a particle in a gauge field

The line element (1) implies the action

$$S=-m\int d\tau\left(-\dot{X}^{2}\right)^{\frac{1-b}{2}}\left(-\alpha\cdot\dot{X}\right)^{b}. \tag{2}$$

If $\alpha\cdot\alpha=0$, this action induces the dispersion relation

$$p^{2}+m^{2}\left(1-b^{2}\right)\left(\frac{-\alpha\cdot p}{m(1-b)}\right)^{\frac{2b}{1+b}}=0. \tag{3}$$

In this work, an arbitrary vector α_{μ} will be taken.

The action (2) has the following constants: m,α_{μ},b. The case $b=0$ gives the usual relativistic particle action and $b=1$ gives

$$S=m\int d\tau\alpha\cdot\dot{X}, \tag{4}$$

which does not have dynamic. For those reasons, we will take $b \neq 0$ and $b \neq 1$. Notice that the cases $m = 0$ or $\alpha_\mu = 0$ can not be studied in the action (2).

The action (2) is equivalent to

$$
\begin{aligned}
S &= \int d\tau \left(\frac{-\dot{X}^2}{\lambda_1} + \frac{-\alpha \cdot \dot{X}}{\lambda_2^{\frac{1-b}{2b}}} + \lambda_1 \lambda_2 \left(\frac{-m}{a} \right)^{\frac{2}{1-b}} \right), \qquad (5) \\
a &= 2 \left(\frac{1-b}{2b} \right)^{b} + \left(\frac{1-b}{2b} \right)^{b-1}, \qquad (6)
\end{aligned}
$$

where λ_1 and λ_2 are Lagrange multipliers. In this alternative action the cases $m = 0$ and $\alpha_\mu = 0$ can be studied.

If $\alpha_\mu = 0$ we get

$$
S = \int d\tau \left(\frac{-\dot{X}^2}{\lambda_1} + \lambda_1 \lambda_2 \left(\frac{-m}{a} \right)^{\frac{2}{1-b}} \right) \qquad (7)
$$

and the Euler-Lagrange equation to λ_2 is

$$
\lambda_1 \left(\frac{-m}{a} \right)^{\frac{2}{1-b}} = 0. \qquad (8)
$$

In this case, the action (7) is equivalent to

$$
S = \int d\tau \frac{-\dot{X}^2}{\lambda_1}, \qquad (9)
$$

which is the usual massless particle action.

When $m = 0$ we obtain

$$
S = \int d\tau \left(\frac{-\dot{X}^2}{\lambda_1} + \frac{-\alpha \cdot \dot{X}}{\lambda_2^{\frac{1-b}{2b}}} \right), \qquad (10)
$$

which is invariant under

$$
X^\mu \to \Lambda X^\mu, \qquad \lambda_1 \to \Lambda^2 \lambda_1, \qquad \lambda_2 \to \Lambda^{\frac{2b}{1-b}} \lambda_2, \qquad (11)
$$

where Λ is a constant. In this case the system has more symmetries. The Euler-Lagrange equation for λ_1, λ_2 are

$$\dot{X}^2 = 0, \qquad \alpha \cdot \dot{X} = \dot{X}^P = 0. \tag{12}$$

Then, the particle travels with velocity of light, but it does not travel in the direction X^P, which means that massless particles only travel in effective $(2+1)$-dimensional space-time. Now, at high energy regime p is bigger than m, then we can take $m \approx 0$. Then at Planck scale it is correct to take $m = 0$, because it is a very energetic regime. For this reason, at this regime all particles are $(2+1)$-dimensional. This is a remarkable result, in fact there are some approaches to quantum gravity with dynamical dimensional reduction at Planck scale [37, 38, 39].

2.1 Gauge symmetry

The action (5) is invariant under the transformation

$$\alpha_\mu \qquad \rightarrow \qquad \alpha_\mu + \lambda_2^{\frac{1-b}{2b}} \frac{\partial \chi}{\partial X^\mu}, \tag{13}$$

where χ is an arbitrary function of space-time. This transformation can be interpreted as a gauge symmetry. This result allows to write the action (5) as a particle in a background gauge field. In fact, using

$$A_\mu = \alpha_\mu, \tag{14}$$

the action (5) becomes

$$S = \int d\tau \left(\frac{-\dot{X}^2}{\lambda_1} - q_{eff} A_\mu \dot{X}^\mu + \lambda_1 \frac{m_{eff}^2}{4} \right) \tag{15}$$

here

$$m_{eff}^2 = 4\lambda_2 \left(\frac{-m}{a} \right)^{\frac{2}{1-b}}, \qquad q_{eff} = \lambda_2^{\frac{b-1}{2b}}. \tag{16}$$

Then, the action (5) is equivalent to

$$S \;\; = \;\; -\int d\tau \left(m_{eff} \sqrt{-\dot{X}^2} + q_{eff} A_\mu \dot{X}^\mu \right), \tag{17}$$

which looks like an action of a particle in a background gauge field A_μ, where mass m_{eff} and charge e_{eff} are function of τ.

3 Gauge field, restoring Lorentz symmetry and a general action

Since $A_\mu = \alpha_\mu$ is a constant gauge field, it does not have dynamics. This field is a particular case of a general vector $B_\mu(X)$, then a generalized VRS action is given by

$$S = \int d\tau \left(\frac{-\dot{X}^2}{\lambda_1} + \frac{-B \cdot \dot{X}}{\lambda_2^{\frac{1-b}{2b}}} + \lambda_1 \lambda_2 \left(\frac{-m}{a} \right)^{\frac{2}{1-b}} \right). \tag{18}$$

If $B_\mu(X)$ is a vector under Lorentz transformations, this symmetry is restored in (18). When $B_\mu(X) = \alpha_\mu$, the Lorentz symmetry is broken and the action (18) reduces to VSR action (2).

When $m = 0$, we obtain

$$S = \int d\tau \left(\frac{-\dot{X}^2}{\lambda_1} + \frac{-B \cdot \dot{X}}{\lambda_2^{\frac{1-b}{2b}}} \right). \tag{19}$$

In this case the Euler-Lagrange equation for λ_1, λ_2 are

$$\dot{X}^2 = 0, \qquad B \cdot \dot{X} = \dot{X}^P = 0. \tag{20}$$

This implies that the particle travels with velocity of light, but it does not travel in the direction X^P, which depends on B_μ. Then the dynamical dimensional reduction is ruled by the field B_μ. However, when $b \to 0$ this field disappears in the action (19) and the particle becomes $(3+1)$-dimensional.

The new action (18) is invariant under gauge transformation

$$B'_\mu(X) = B_\mu(X) + \lambda_2^{\frac{1-b}{2b}} \frac{\partial \chi}{\partial X^\mu}. \tag{21}$$

Using B_μ, it is possible construct a covariant derivative in the following way

$$D_\mu = \partial_\mu + i \lambda_2^{\frac{b-1}{2b}} B_\mu, \tag{22}$$

which gives the field strength tensor

$$[D_\mu, D_\nu] = i \lambda_2^{\frac{b-1}{2b}} F_{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{23}$$

If VSR is important at Planck scale, $F_{\mu\nu}$ might be important at that regime too. However, $F_{\mu\nu}$ vanish when $b \to 0$.

Now, if we take

$$\lambda_2^{\frac{1-b}{2b}} = \frac{\lambda_1}{2\zeta} \tag{24}$$

we get

$$\begin{aligned} S &= \int d\tau \left(\frac{-1}{\lambda_1} \left(\dot{X}^2 + 2\zeta \dot{X} \cdot B \right) + \frac{\lambda_1^{\frac{1+b}{1-b}}}{(2\zeta)^{\frac{2b}{1-b}}} \left(\frac{-m}{a} \right)^{\frac{2}{1-b}} \right) &(25) \\ &= \int d\tau \left(\frac{-1}{\lambda_1} \left(\dot{X}_\mu + \zeta B_\mu \right)^2 + \frac{\zeta^2 B^2}{\lambda_1} + \frac{\lambda_1^{\frac{1+b}{1-b}}}{(2\zeta)^{\frac{2b}{1-b}}} \left(\frac{-m}{a} \right)^{\frac{2}{1-b}} \right). &(26) \end{aligned}$$

This action is a particular case of

$$S = \int d\tau \left(\frac{-1}{\lambda_1} \left(\dot{X}_\mu + \zeta B_\mu \right)^2 - \lambda_1 D B^2 + \frac{\lambda_1^{\frac{1+b}{1-b}}}{(2\zeta)^{\frac{2b}{1-b}}} \left(\frac{-m}{a} \right)^{\frac{2}{1-b}} \right), \tag{27}$$

in fact, if we take

$$D = -\frac{\zeta^2}{\lambda_1^2}, \tag{28}$$

we obtain (26). Then the action (26) is more general than the action for very special relativity and its symmetries depend on B_μ.

We saw that if $m = 0$ the action (5) has more symmetries. Now, when we take $m = 0$ in (26), we get

$$S = \int d\tau \left(\frac{-1}{\lambda_1} \left(\dot{X}_\mu + \zeta B_\mu \right)^2 - \lambda_1 D B^2 \right), \tag{29}$$

which is invariant under scale transformation

$$X^\mu \to \Lambda X^\mu, \quad B^\mu \to \Lambda B^\mu, \quad \lambda_1 \to \Lambda^2 \lambda_1, \quad D \to \Lambda^{-4} \lambda_1, \quad \zeta \to \zeta. \tag{30}$$

4 Two-time physics

X^μ is a natural vector field under Lorentz transformation. For this field, we have

$$S = \int d\tau L = \int d\tau \left(\frac{-1}{\lambda_1} \left(\dot{X}_\mu + \zeta X_\mu \right)^2 - \lambda_1 D X^2 \right), \tag{31}$$

which is invariant under

$$X^\mu \to \Lambda X^\mu, \quad \lambda_1 \to \Lambda^2 \lambda_1, \quad D \to \Lambda^{-4} \lambda_1, \quad \zeta \to \zeta. \tag{32}$$

The action (31) is equivalent to two-time physics action, to show this statement notice that

$$\begin{aligned}
P_\mu &= \frac{\partial L}{\partial \dot{X}^\mu} = \frac{-2}{\lambda_1} \left(\dot{X}_\mu + \zeta X_\mu \right), \\
P_{\lambda_1} &= \frac{\partial L}{\partial \dot{\lambda}_1} = 0, \\
P_\gamma &= \frac{\partial L}{\partial \dot{\zeta}} = 0, \qquad (33) \\
P_D &= \frac{\partial L}{\partial \dot{D}} = 0. \qquad (34)
\end{aligned}$$

Then the canonical Hamiltonian is

$$H_c = -\frac{\lambda_1}{4} P^2 - \zeta P \cdot X + \lambda_1 D X^2, \tag{35}$$

meanwhile the total Hamiltonian is [46]

$$H_T = -\frac{\lambda_1}{4} P^2 - \zeta P \cdot X + \lambda_1 D X^2 + h_1 P_{\lambda_1} + h_2 P_\zeta + h_3 P_D, \tag{36}$$

where h_1, h_2, h_3 are Lagrange multipliers. Therefore, using Dirac's method [46] we find

$$\begin{aligned}
\dot{P}_{\lambda_1} &= \{P_{\lambda_1}, H_T\} = \left(\frac{P^2}{4} - D X^2 \right) \approx 0, \\
\dot{P}_\zeta &= \{P_\zeta, H_T\} = P \cdot X \approx 0, \qquad (37) \\
\dot{P}_D &= \{P_D, H_T\} = \lambda_1 X^2 \approx 0, \qquad (38)
\end{aligned}$$

which implies the first class constraints

$$
\begin{aligned}
\phi_1 &= \frac{P^2}{2} \approx 0, \\
\phi_2 &= P \cdot X \approx 0, && (39)\\
\phi_3 &= \frac{X^2}{2} \approx 0. && (40)
\end{aligned}
$$

With these constraints we obtain the extended Hamiltonian [46]

$$H_E = H_T + \beta_1 \phi_1 + \beta_4 \phi_2 + \beta_5 \phi_3, \tag{41}$$

it can be written as

$$H_E = \gamma_1 \phi_1 + \gamma_2 \phi_2 + \gamma_3 \phi_3, \tag{42}$$

where

$$\gamma_1 = \left(\beta_1 - \frac{\lambda_1}{2}\right), \qquad \gamma_1 = (-\zeta + \beta_2), \qquad \gamma_3 = (2\lambda_1 D + \beta_3). \tag{43}$$

Expression (42) is the two-time physics Hamiltonian which is invariant under local $SL(R,2)$ and global conformal group $SO(2,d)$ [43, 42], another work about this system can be found in [47].

Notably two-time physics acts as a model that unifies dynamics of different systems [43]. In particular, this theory contains the relativistic free particle, the non-relativistic particle and other particles [43]. Then, when the Lorentz symmetry is restored in the action (27), a unified model is obtained. This result allows conjecture that if Lorentz symmetry is broken at Planck scale, when this symmetry is restored a unified model is obtained. A work about two-time physics and breaking Lorentz symmetry can be seen in [44]. It is interesting to mention that recently have been proposed optical metamaterials with two-time properties [45].

5 Summary

In this work the action for a particle in very special relativity was studied. It was shown that this action can be written as an action for a relativistic

particle in a background gauge field. This gauge field causes that the massless particles become $(2+1)$-dimensional. Now, at Planck scale the particles are very energetic and we can take $m=0$, for all particles. Then, at this regime all particles are $(2+1)$-dimensional. This is a remarkable result, in fact there are some approaches to quantum gravity with dynamical dimensional reduction at Planck scale [37, 38, 39]. In VSR the dimensional reduction is ruled by the gauge field, which disappears in the limit $b \to 0$. In that limit all particles become $(3+1)$-dimensional. Furthermore, the Lorentz symmetry was restored and generalized action was obtained. It was shown that this new action contains very special relativity and two-time physics. It is worth notice that two-time physics acts as a model that unifies dynamics of different systems [43]. This last result allows conjecture that if Lorentz symmetry is broken at Planck scale, when this symmetry is restored a unified model is obtained. Then, it is possible that at Planck scale the Lorentz symmetry is broken and there is dimensional reduction, but at the regime where the Lorentz symmetry is restored a unified model is obtained.

References

[1] P. Horava, Phys. Rev. D **79**, 084008 (2009) .

[2] D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998).

[3] D. Anselmi, Phys. Rev. D **79**, 025017 (2009).

[4] D. Anselmi, M. Taiuti, Phys. Rev. D **81**, 085042 (2010).

[5] F. Girelli, S. Liberati and L. Sindoni, Phys.Rev. D **75**, 064015 (2007).

[6] A. G. Cohen and S. L. Glashow, Phys. Rev. Lett. **97**, 021601 (2006).

[7] A.P. Kouretsis, M. Stathakopoulos and P.C. Stavrinos, Phys. Rev. D **79**, 104011 (2009).

[8] M. M. Sheikh-Jabbari and A. Tureanu, Phys. Rev. Lett. **101**, 261601 (2008).

[9] S. Das, S. Ghosh and S. Mignemi, Phys.Lett. A **375**, 3237 (2011).

[10] G. W. Gibbons, J. Gomis and C. N. Pope, Phys. Rev. D **76**, 081701 (2007).

[11] G.Y. Bogoslovsky and H.F. Goenner, Phys. Lett. A **323**, 40 (2004).

[12] E. Kiritsis, *Lorentz violation, Gravity, Dissipation and Holography,* arXiv:1207.2325 [hep-th].

[13] V. A. Kostelecky and R. Lehnert, Phys. Rev. D **63**, 065008 (2001).

[14] V. A. Kostelecky, R. Lehnert and M. J. Perry Phys. Rev. D **68**, 123511 (2003).

[15] R. Lehnert, Phys. Rev. D **68**, 085003 (2003).

[16] V. A Kostelecky and N. Russell, Phys. Lett. B **693**, 443 (2010).

[17] G. Cacciapaglia, A. Deandrea and L. Panizzi, JHEP **1111**, 137 (2011).

[18] X-J. Bi, P-F. Yin, Z-H Yu and Q. Yuan, Phys. Rev. Lett. **107**, 241802 (2011).

[19] S. S. Gubser, Phys. Lett. B **705**, 279 (2011).

[20] S. Nojiri and S. D. Odintsov, Eur. Phys. J. C **71**, 1801 (2011).

[21] G. Amelino-Camelia, G. Gubitosi, N. Loret, F. Mercati, G. Rosati and P. Lipari, Int. J. Mod. Phys. D **20**, 2623 (2011).

[22] G. Dvali and A. Vikman, JHEP **02**, 134 (2012).

[23] J. Alexandre, J. Ellis and N. E. Mavromatos, Phys.Lett. B **706**, 456 (2012).

[24] J. M. Romero, J. A. Santiago and O. Gonzalez-Gaxiola, Mod. Phys. Lett. A **27**, 1250060 (2012).

[25] V. Baccetti, K. Tate and M. Visser, JHEP **1205**, 119 (2012).

[26] G. F. Giudice, S. Sibiryakov and A. Strumia, Nucl.Phys. B861 (2012) 1-16.

[27] F. R. Klinkhamer, Phys. Rev. D **85**, 016011 (2012).

[28] E. N. Saridakis, *Superluminal neutrinos in Horava-Lifshitz gravity,* e-Print: arXiv:1110.0697 [gr-qc].

[29] A. G. Cohen, JHEP **0707**, 039 (2007).

[30] S. Petras, R. von Unge and J. Vohanka, JHEP **1107**, 015 (2011).

[31] S. Ghosh, P. Pal Phys.Rev. D **80**, 125021 (2009).

[32] S. Cheon, C. Lee and S. J. Lee, Phys. Lett. B **679**, 73 (2009).

[33] D.V. Ahluwalia and S.P. Horvath, JHEP **1011**, 078 (2010).

[34] S. Das and S. Mohanty, Mod.Phys.Lett. A **26**, 139 (2011).

[35] W. Muck, Phys. Lett. B **670**, 95 (2008).

[36] E. Alvarez and R. Vidal, Phys .Rev. D **77**, 127702 (2008).

[37] J. Ambjørn, J. Jurkiewicz, and R. Loll, Phys. Rev. Lett. **95**, 171301 (2005).

[38] P. Hořava, Phys. Rev. Lett. **102**, 161301 (2009).

[39] G. Giasemidis, J. F. Wheater, S. Zohren, Phys. Rev. D **86**, 081503(R) (2012).

[40] R. Marnelius and B. Nilsson, Phys. Rev. D **20**, 839 (1979).

[41] R. Marnelius, Phys. Rev. D **20**, 2091 (1979).

[42] J. M. Romero and A. Zamora, Phys. Rev. D **70**, 105006 (2004).

[43] I. Bars, Int.J.Mod.Phys. A **25** , 5235 (2010).

[44] J. M. Romero, O. Sanchez-Santos and J. D. Vergara, Phys. Lett. A **375**, 3817 (2011).

[45] I. I. Smolyaninov and E. E. Narimanov, Phys. Rev. Lett. **105**, 067402 (2010).

[46] P. A. M. Dirac, *Lectures on Quantum Mechanics,* Dover, New York (2001).

[47] C. Leiva and M. S. Plyushchay, Annals Phys. **307**, 372 (2003).